Analysis of Jet Aircraft Engine Exhaust Nozzle Entrance Profiles, Accountability, and Effects

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Methods of averaging total pressure and temperature profiles at the entrance of jet engine exhaust nozzles were evaluated analytically. The concept of "conservation of ideal available thrust" was used to determine the best averaging technique. Results show that pressure profiles should be mass-weighted and temperature profiles should be "thrust"-weighted to determine properly the actual ideal thrust available to the nozzle. A brief analysis of profiles effects on converging-diverging (CD) nozzle performance was conducted using both analytical and experimental approaches. Results indicated that performance is unaffected by the presence of entrance profiles, provided that they are accounted for properly.

Nomenclature

A = flow area
α = flow angle relative to nozzle centerline
C_{fg} = nozzle thrust coefficient
C_p = specific heat of gas at constant pressure
C_v = nozzle velocity coefficient
D = diameter
δ = boundary-layer thickness
δ^* = boundary-layer displacement thickness
F_g = nozzle actual gross thrust
F_{gi} = nozzle ideal gross thrust based on isentropic expansion
F_s = total momentum of gas flow
g = gravity constant
γ = ratio of specific heats of gas
H = enthalpy based on total conditions
h = enthalpy based on static conditions
J = Joule's constant
L = nozzle length
M = Mach number
$\dot{m}$ = static flow function
P = gas pressure
R = gas constant
ρ = gas density
T = gas temperature
θ = boundary-layer momentum thickness
V = gas velocity
W = weight flow

Subscripts

j = counting index at station 7
0 = ambient conditions
s = isentropic expansion to ambient conditions
t = total gas conditions
∞ = freestream conditons
7 = nozzle entrance, station 7
8 = nozzle throat, station 8
9 = nozzle exit, station 9

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Introduction

INTERNAL performance for jet engine exhaust nozzles has been predicted historically by a combination of analytical design studies and scale-model tests. These tests can be conducted under precisely controlled conditions, including accurate knowledge of nozzle entrance flow conditions.

Real full-scale engine performance tests, however, cannot provide these precise controls, and nozzle entrance conditions are known less precisely. Specifically, a good scale-model test is conducted with very low-distortion total pressure and temperature profiles [$(P_{t\max}-P_{t\min})/P_{t\max}<1\%$ and $(T_{t\max}-T_{t\min})<2°F$]. Uniform profiles on real full-scale hardware, however, seldom are encountered because of the performance of upstream component hardware such as turbines, turbine frames and centerbodies, afterburners, bypass engine mixers, etc.

This paper deals with the questions of how to account for these profiles in nozzle performance evaluation, and what is the effect of these profiles on nozzle performance. Additional information may be found in Ref. 1. Accounting of profiles was performed analytically by considering a number of averaging techniques for a wide range of nozzle entrance profile shapes and combinations. A rudimentary assessment of profile effects on converging-diverging (CD) nozzle performance was done both analytically and by scale-model test.

Analysis of Profile-Averaging Methods

There are numerous ways of averaging profile data to arrive at a single, one-dimensional value of P_t or T_t. Some involve weighting factors (i.e., area or mass weighting), and some involve the conservation of some quantity (i.e., energy or momentum). The particular method chosen should be one that best suits the application and the objectives. For exhaust nozzles, the objective is to arrive at a total pressure- and/or temperature-averaging procedure that can be used in evaluating nozzle performance. Although both flow and thrust coefficients are important nozzle performance parameters, the flow coefficient is used to size a nozzle to an engine cycle, whereas the thrust coefficient is the measure of nozzle efficiency with which it expands the gas flow. It is the thrust coefficient that was addressed in this analysis.

The definition of thrust coefficient as it commonly is used is

$$C_{fg}=F_g/F_{gi} \tag{1}$$

where F_{gi} is based on the actural weight flow expanded isentropically to ambient pressure.

Thus, the ideal thrust available to the nozzle is the basis upon which nozzle performance is evaluated. If profiles exist and the ideal thrust of each streamtube of flow entering the nozzle were calculated and integrated over the profile, the integral then would represent the total, exact ideal thrust available to the nozzle. This is the thrust upon which nozzle performance should be based because it represents the real ideal thrust available to the nozzle. Therefore, the objective of a profile-averaging procedure should be such that, when the averaged flow properties (P_t and T_t) are used to calculate ideal thrust based on actual total weight flow, the ideal thrust calculated is equal to that which would be obtained by integrating the profile, i.e.

$$F_{gi_{\text{av properties}}} = \int_{\text{profile}} F_{gi} \tag{2}$$

Equation (2), which can be termed "conservation of ideal available thrust," is the basis upon which various averaging techniques were evaluated. The total exact ideal thrust was calculated by integration for a wide range of profiles. The ideal thrust based on the averaged properties then was calculated and compared to the integrated values.

Selection of Sample Profiles

The selection of sample nozzle inlet total pressure and temperature profiles was intended to cover typical regimes of exhaust nozzle operation for most present and near-future jet aircraft engine applications. These include the non-afterburning mixed and separate flow turbofan engines typical of subsonic transports and the afterburning engines for military application with both subsonic and supersonic design points. Variables included nozzle entrance duct Mach number, overall nozzle pressure ratio, total pressure profile distortion (percent), and total temperature profile distortion (ΔT_t in degrees).

The profiles were assumed to occur at the nozzle entrance, station 7, as shown in Fig. 1 for a typical CD nozzle. To simplify the calculations, the static pressure across the duct was assumed to be constant, air was assumed for all gas properties, and the duct was divided into three sections of equal flow area with different total pressures and temperatures assigned to each section. A total of 33 combinations of the following variable ranges were evaluated: duct Mach number, $M_7 = 0.3$ and 0.6; nozzle pressure ratio, $P_t/P_0 = 2.5$ and 10; P_t distortion $= \pm 3.1$, 6.4, and 13.6%; and T_t distortion $= \pm 300$, 600, and 1200°F.

Calculation of Real Ideal Thrust

The real available ideal thrust of the flow entering the nozzle is equal to the summation of the ideal thrust of each of the three streamtubes. This thrust was calculated exactly for each sample profile case chosen and was then the baseline thrust to which all ideal thrusts calculated from the various averaging techniques were compared.

All thrusts were calculated on an ideal basis assuming complete isentropic expansion of the airflow to ambient pressure. The equation for the thrust can be derived from the energy equation and is represented as follows

$$F_{gi_{\text{profile}}} = \sum_{j=1}^{3} F_{gi} = \sum_{j=1}^{3} W_j \sqrt{(2J/g)\,(H_7 - h_S)_j} \tag{3}$$

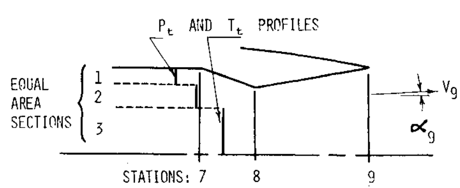

Fig. 1 Converging-diverging nozzle schematic.

The weight flow W_j was calculated from the following continuity equation at station 7

$$W_j = \sqrt{\gamma g/RT_{tj}}\;\; PM_j\sqrt{1 + [(\gamma - 1)/2]M_j^2}\; A_j \tag{4}$$

where

$$M_j = \sqrt{[2/(\gamma - 1)]\,[(P_{tj}/P)^{(\gamma - 1)/\gamma} - 1]} \tag{5}$$

For the case of the averaged properties, Eq. (3) becomes

$$F_{gi_{\text{av}}} = \sqrt{(2J/g)\,(H_7 - h_s)_{\text{av}}}\sum_{j=1}^{3} W_j \tag{6}$$

where $(H_7 - h_s)_{\text{av}}$ is based on $P_{t\text{av}}$ and $T_{t\text{av}}$.

Evaluation and Selection of Profile-Averaging Methods

With the real, available ideal thrust of the profiles established, various methods of averaging the total pressure and temperature were investigated to determine which one yields an ideal thrust that is closest to the real ideal thrust. Evaluation of the averaging methods was done by applying them first to a total pressure profile only and then to a total temperature profile only. Procedures that appeared promising then were applied to profiles with both pressure and temperature distortion.

Method 1: Area Weighting

This is a simple area-weighted averaging technique where total pressure and temperature are obtained as follows:

$$T_{t_{\text{av}}} = \sum A_j T_{tj} / \sum A_j,\;\; P_{t\text{av}} = \sum A_j P_{tj} / \sum A_j \tag{7}$$

Method 2: Mass Weighting

This procedure attempts to account for flow distribution by using flow as an averaging or weighting factor. Thus

$$T_{t\text{av}} = \sum W_j T_{tj} / \sum W_j,\;\; P_{t\text{av}} = \sum W_j \tag{8}$$

Note that, if C_p is assumed constant, the temperature average represents a conservation-of-energy approach.

Method 3: Conservation of Energy

This approach assumes that the total energy of the flow is conserved and is derived from the energy equation as follows

$$W_{\text{total}} H_{\text{av}} = \sum W_j H_j \tag{9}$$

$$H_{\text{av}} = \sum W_j H_j / \sum W_j \tag{10}$$

A total energy level is conserved in this manner, and an average total temperature level is established. Since enthalpy is primarily a function of temperature, this approach is not used easily as an averaging process for total pressure.

Method 4: Conservation of Mass and Momentum

This approach requires that the averaged, one-dimensional flowstream properties be such that the mass flow and total momentum at the nozzle entrance are conserved. The procedure requires the simultaneous solution of the continuity and momentum equations. The continuity equation is identical to Eq. (4) and is rearranged as follows with slightly different subscripts

$$W_{\text{total}}\sqrt{R_{\text{av}} T_{t\text{av}} / \gamma_{\text{av}} g} = P_{\text{av}} M_{\text{av}} A_{\text{total}} \sqrt{1 + [(\gamma_{\text{av}} - 1)/2] M_{\text{av}}^2} \tag{11}$$

where

$$W_{\text{total}} = \sum W_j$$

The total momentum equation F_s is represented as follows.

$$F_s = PA + (W/g)V = (1+\gamma M^2)PA \tag{12}$$

The total integrated momentum of the actual profile is, therefore

$$F_{s7} = \sum (1+\gamma_j M_j^2)P_j A_j \tag{13}$$

This must equal the total momentum of the averaged stream; thus

$$F_{s7} = (1+\gamma_{\text{av}} M_{\text{av}}^2)P_{\text{av}} A_{\text{total}} \tag{14}$$

If Eq. (11) is divided by Eq. (14), we have

$$\frac{W_{\text{total}}\sqrt{(RT_{t\text{av}}/\gamma_{\text{av}}g}}{F_{s7}} = \frac{\sqrt{1+[(\gamma_{\text{av}}-1)/2]M_{\text{av}}^2}}{(1+\gamma_{\text{av}}M_{\text{av}}{}^2)} \tag{15}$$

W_{total} and F_{s7} are the actual weight flow and total momentum as calculated (integrated) from the known profiles. This approach was evaluated with the pressure profiles only, and all of the quantities on the left-hand side of Eq. (15) can be calculated from the profile. Thus, Eq. (15) can be used to solve for M_{av} by iteration. With the average Mach number established, the new, average static pressure is calculated from Eq. (14). Note that, even for a constant static pressure case, the calculated average static pressure will differ from the real pressure because the calculated pressure is really an equivalent pressure required to satisfy conservation of mass and momentum. With average (equivalent) static pressure and duct Mach number established, the total pressure can be calculated using the usual isentropic relations.

The resulting averaged (or equivalent) quantities, $P_{t\text{av}}$, P_{av}, and M_{av}, when combined with the average total temperature and the total flow area, will provide the same mass flow and total momentum as the real profile. For cases of both pressure and temperature profiles, this approach could be combined with method 3 to yield a conservation of mass, momentum, and energy approach.

The techniques described thus far attempt to conserve one or more of the basic fluid flow quantities (mass, momentum, and energy) or use weighting factors such as mass flow and area. However, if we address ourselves to the earlier discussion, a logical approach would seem to be one that satisfies "conservation of ideal available thrust." The enthalpy drop $H_7 - h_s$ in Eq. (3) is a function of both total temperature and total and ambient pressure. Since the two quantities of interest (P_t and T_t) strongly influence the enthalpy drop, a good averaging procedure would be one that contains some form of Eq. (3). Equation (2) can be expanded to obtain

$$(W_{\text{total}}/g)\sqrt{2gJ(H_7-h_s)_{\text{av}}} = \sum (W_j/g)\sqrt{2gJ(H_7-h_s)_j} \tag{16}$$

Rearranging and simplifying gives

$$(H_7 - h_2)_{\text{av}} = \left[\sum W_j\sqrt{(H_7-h_s)_j}/W_{\text{total}}\right]^2 \tag{17}$$

By definition, the ΔH_{av} calculated from Eq. (17) will provide the exact same thrust as the integrated profile. The following methods address themselves to this general approach.

Method 5: Conservation of Thrust

An average enthalpy drop is calculated based on Eq. (17) and is rewritten as follows in simplified form.

$$\Delta H_{\text{av}} = \left[\frac{\sum W_j\sqrt{\Delta H_j}}{\sum W_j}\right]^2 \tag{18}$$

The ΔH_{av} calculated from this equation will give, by definition, the exact same calculated thrust as the integrated profile; however, this method does not provide an average total pressure or temperature directly and is not necessarily convenient.

Method 6: Conservation of Thrust, Enthalpy Approximation 1

Method 5 can be modified by assuming a constant specific heat for the expansion process. The isentropic exhaust gas velocity can be written using the isentropic relation

$$T/T_t = (P/P_t)^{(\gamma-1)/\gamma}$$

as

$$V_s = \sqrt{2gJC_pT_t\left[1-(P_0/P_t)^{(\gamma-1)/\gamma}\right]} \tag{19}$$

Substituting $H = C_pT_t$, the equation for thrust is

$$F_{gi} = \frac{W}{g}V_s = \frac{W}{g}\sqrt{2gJH\left[1-(P_0/P_t)^{(\gamma-1)/\gamma}\right]} \tag{20}$$

Rearranging

$$\frac{F_{gi}}{W\sqrt{H}} = \sqrt{(2J/g)\left[1-(P_0/P_t)^{(\gamma-1)/\gamma}\right]} = \bar{F}' \tag{21}$$

The ideal thrust of a stream thus will be

$$F_{gi} = \bar{F}'\, W\sqrt{H} \tag{22}$$

If the appropriate subscripts are substituted into Eq. (2), we obtain

$$\bar{F}_{\text{av}} W_{\text{total}}\sqrt{H_{\text{av}}} = \sum \bar{F}_j' W_j\sqrt{H_j} \tag{23}$$

Rearranging and solving for H_{av}, we have finally

$$H_{\text{av}} = \left[\frac{\sum \bar{F}' W_j\sqrt{H_j}}{\bar{F}_{\text{av}}\sum W_j}\right]^2 \tag{24}$$

Where $\bar{F}_j'$ is calculated using Eq. (21) for each element of the profile, knowing P_t and $T_t[\gamma = f(T_t)]$; and $\bar{F}_{\text{av}}$ is based on using Eq. (21), where $P_{t\text{av}}$ is calculated from a supplemental method, i.e., mass-averaged, and

$$\gamma_{\text{av}} = \sum W_j\gamma_j / \sum W_j$$

as a first approximation. This method requires an iteration for $\bar{F}'_{\text{av}}$ and $T_{t\text{av}}$ and is cumbersome.

Method 7: Conservation of Thrust, Enthalpy Approximation 2

If total pressure profiles are small, the equation for the average total enthalpy can be simplified greatly by eliminating the thrust function approximation $\bar{F}'$. With this modification, the average total enthalpy (and thus average total temperature) can be obtained from the following equation.

$$H_{av} = \left[\frac{\sum W_j (\sqrt{H_j})}{\sum W_j} \right]^2 \tag{25}$$

Methods 5-7 have addressed themselves to some form of enthalpy averaging, which indirectly arrives at an average temperature. With slight modifications, these approaches can be made to calculate a thrust-averaged temperature directly.

Method 8: Conservation of Thrust, Temperature Approximation 1

Using Eq. (19) for the isentropic velocity, the equation for thrust is written as follows.

$$F_{gi} = \frac{W}{g} V_s = \frac{W}{g} \sqrt{2gJC_p T_t \left[1 - (P_0/P_t)^{(\gamma-1)/\gamma} \right]} \tag{26}$$

Substituting $C_p = \gamma R/(\gamma-1)J$ and rearranging yields

$$\frac{F_{gi}}{W\sqrt{T_t}} = \sqrt{\frac{2\gamma R}{g(\gamma-1)} \left[1 - \left(\frac{P_0}{P_t} \right)^{(\gamma-1)/\gamma} \right]} = \bar{F} \tag{27}$$

The ideal thrust of a stream thus will be

$$F_{gi} = \bar{F} W \sqrt{T_t} \tag{28}$$

Substituting the appropriate subscripts into Eq. (2) and rearranging in a similar manner as in method 6, we have

$$T_{tav} = \left[\frac{\sum \bar{F}_j W_j \sqrt{T_{tj}}}{\bar{F}_{av} \sum W_j} \right]^2 \tag{29}$$

where $\bar{F}_j$ is calculated using Eq. (27) for each element of the profile, knowing P_t and T_t $[\gamma = f(T_t)]$; and $\bar{F}_{av}$ is based on P_{tav} calculated from a supplemental method as in method 6, and

$$\gamma_{av} = \sum W_j \gamma_j / \sum W_j$$

as a first approximation. As in method 6, this approach requires an iteration for $\bar{F}_{av}$ and T_{tav}.

Method 9: Conservation of Thrust, Temperature Approximation 2

Similar to the approach for method 7, Eq. (29) can be simplified by eliminating the thrust function in the average. The resulting equation is a very simple square mean root mass weighting of the temperature

$$T_{tav} = \left[\frac{\sum W_j (\sqrt{T_{tj}})}{\sum W_j} \right]^2 \tag{30}$$

The preceding averaging methods were applied to selected pressure profile cases and temperature profile cases. For the total pressure profiles, methods 1, 2, and 4 were used. For the total temperature profiles, all methods but No. 4 were used.

Results of this analysis are presented as a ratio of the ideal thrust calculated by the averaged quantities divided by the actual ideal thrust obtained from the integrated calculations in

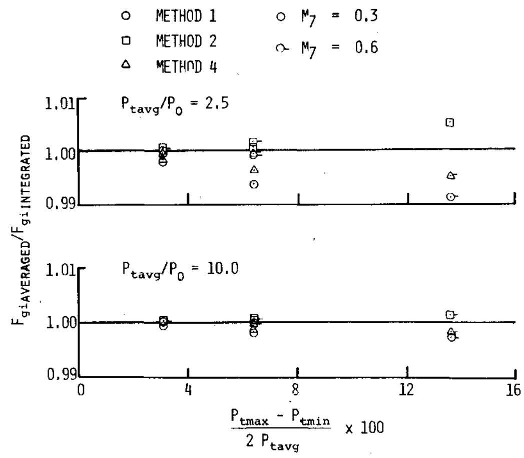

Fig. 2 Results of pressure-averaging methods.

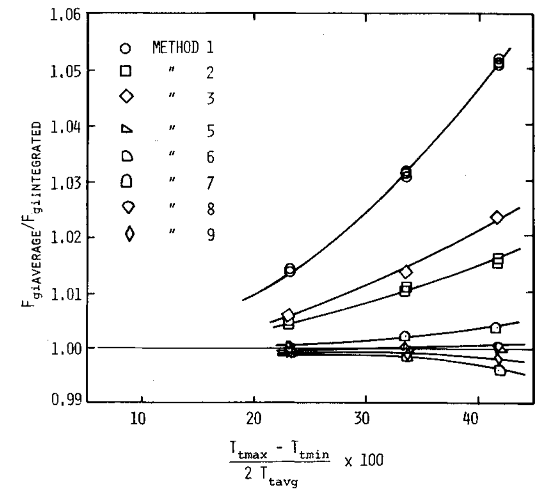

Fig. 3 Results of temperature-averaging methods.

Figs. 2 and 3. The ratios are plotted vs percent profile distortion.

Inspection of Fig. 2 indicates that the lower nozzle pressure ratios are more sensitive to errors than the high pressure ratios. This is not unexpected, since, for a given mass flow and total temperature, nozzle thrust changes more rapidly at low pressure ratios than at high pressure ratios. The area-weighting technique is the least accurate of the three methods, and the mass-weighting and conservation of mass and momentum methods give approximately the same errors, only in opposite directions. Since mass weighting is much simpler and easier than the conservation of mass and momentum approach, this method would be preferred. For most typical levels of pressure distortion in nozzles ($\leq \pm 6\%$), the error in ideal thrust calculation would be less than 0.15%.

Results in Fig. 3 show that again the area-weighting technique is the least accurate method and can result in errors of 5% at high levels of distortion (typical for part-reheat operation). Methods 2 and 3, mass weighting and conservation of energy, provide nearly the same results and differ only by the assumption of a constant specific heat. These two methods are incorrect because, by definition, they assume complete mixing of the flow to obtain an average temperature. It has been demonstrated both analytically and by tests that the mixing of two streams of widely differing temperatures will provide a thrust increase. This thrust augmentation is the reason for incorporating exhaust system mixers in current bypass fan engines. If the stream is, in fact, not fully mixed and the averaging method assumes that it is, then a false impression of a thrust increase will occur, and a high thrust will be calculated, as verified by the results in Fig. 3.

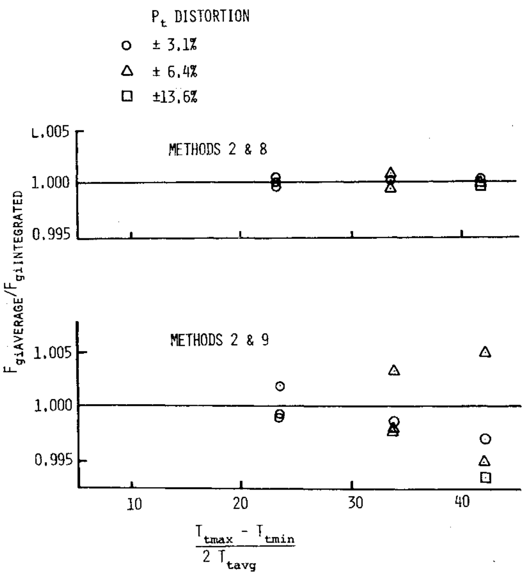

Fig. 4 Results of pressure and temperature profile combinations averaging methods.

Methods 5-9 address themselves to the conservation of thrust general approach. Results show that method 5 is exact, as expected, and that the remaining methods have less than 0.4% error at the extreme profiles. Methods 8 and 9 are easier to handle and are the closest of the conservation-of-thrust methods (<0.17%).

Based on the results in Figs. 2 and 3, combinations of pressure and temperature profiles were evaluated using method 2 for pressure averaging and methods 8 and 9 for temperature averaging. Results are presented in Fig. 4.

The first approach was a combination of methods 2 and 8. The maximum error in ideal thrust calculation was less than 0.1% for all profiles analyzed, as shown in Fig. 4. Using methods 2 and 9, the error has increased to a maximum value of 0.6%. It is important to note that not only is the magnitude of the pressure and temperature distortion significant, but the arrangement by which they are coupled also is significant. Combinations of high pressure with high temperature result in different errors than combinations of the same high pressure with a low temperature. Thus, the knowledge of the distortion levels alone will not allow an error assessment, and, in fact, the error for each profile combination will be unique.

The following conclusions are reached based on this analysis:

1) Averaging of total pressure profiles is performed best by mass weighting. If distortion is less than ±5%, the error in ideal thrust will be less than 0.1% for exhaust nozzle pressure ratios greater than 2.5. If distortion is greater than ±5%, or if pressure ratios are less than 2.5, a thrust calculation adjustment relative to an area-weighted calculation can be estimated easily using the methods herein.

2) Averaging of total temperature profiles should be accomplished by some form of "thrust weighting" in order to minimize thrust calculation errors (less than 0.1%). The form depends on the extent of the profile and the presence and extent of a pressure profile and could be either method 8 or 9.

These techniques should be used whenever nozzle entrance total pressure or temperature profiles exist, in order to assess exhaust nozzle performance properly. Nozzle type or operating condition does not restrict this approach.

Evaluation of Nozzle Entrance Profile Effects

Entrance profile effects on CD nozzle performance were evaluated on a limited basis by two methods:

1) A theoretical analysis was used to evaluate total temperature profiles for two typical operating conditions: a low-temperature, low-distortion profile typical of nonafterburning cruise conditions; and a high-temperature, high-distortion profile indicative of part-afterburning conditions.

2) Experimental analysis of total pressure profile effects was performed using a scale model of a typical CD nozzle. One profile was evaluated typical of nonafterburning cruise conditions.

The conclusions based on these analyses are restricted by the preceding limitations.

Theoretical Approach

The theoretical evaluation of temperature profile effects was conducted by first defining the aerodynamic flowfield within the nozzle for both uniform and nonuniform profiles. This was done by solving the inviscid equations of motion in the nozzle. The nozzle wall boundary-layer parameters then were determined analytically, and an analysis was developed to evaluate the complete viscid flowfields and compare the uniform with the nonuniform cases. The accuracy of the approach was established by comparing a uniform profile theoretical results with scale-model test data.

Nozzle Flowfield Definition

The nozzle internal flowfield was defined using a computer program developed by General Electric under a contract to NASA Langley Research Center.[2] This program, designated the Streamtube Curvature Method (STC), solves the inviscid equations of motion exactly. The method determines the flowfield properties (Mach number, static pressure, and temperature), streamlines, and pressure distributions on bodies for both internal and/or external flows. It can vary the theormodynamic properties of the streams (with the exception of fuel-air ratios), and this multistream capability allows the simulation of a nonuniform velocity profile, as was the case here. Basic program assumptions are thus: inviscid flow, no separation, and no mixing.

Nozzle Flowfield Analysis

The analysis was conducted by calculating the actual nozzle exit thrust based on the flowfield at the nozzle exit, the ideal available thrust based on the entrance flow, and dividing the two to obtain nozzle thrust coefficient. This was done for both uniform and nonuniform profiles for the same nozzle geometry, and the results were compared.

The actual nozzle thrust is defined as the integral of the axial velocity momentum and pressure-area force at the exit plane of the nozzle, station 9, as represented by

$$F_g = \int_{A_9} \frac{dW}{g} V_9 \cos\alpha_9 + \int_{A_9} (P_9 - P_0)\, dA \tag{31}$$

Equation (31) was solved along the orthogonal line at station 9 defined from the STC computer program in the nozzle. All of the aerodynamic properties along the orthogonal are obtained from the program output (i.e., V_9, α_9, P_9, etc.).

The ideal thrust available to the nozzle is obtained with the aid of the appropriate equations and the method 9 averaging techniques, Eq. (30). Thus, the theoretical inviscid nozzle thrust coefficients calculated from the STC program results are obtained using

$$C_{fginviscid} = \left[\int_{A_9} \frac{dW}{g} V_9 \cos\alpha_9 + \int_{A_9} (P_9 - P_0) dA_9\right] \Big/ \left(\int \frac{dW}{g} V_s\right) \tag{32}$$

where

$$V_s = \sqrt{\frac{2\gamma R T_{tav}}{g(\gamma - 1)}\left[1 - \left(\frac{P_0}{P_{tav}}\right)^{(\gamma-1)/\gamma}\right]} \tag{33}$$

The resulting coefficient includes losses in the nozzle due to exit flow angularity ($\cos\alpha_9$ effect) and nozzle under- and overexpansion ($P_9 \neq P_0$). The friction loss in the nozzle was obtained from an estimate of the velocity coefficient. This coefficient is defined in Eq. (34) as the ratio of the actual momentum thrust to the isentropic momentum thrust for the same mass flow.

$$C_V = \int_{A_9} \mathrm{d}WV / \int_{A_9} \mathrm{d}WV_s \tag{34}$$

The only fluid mechanism that acts to retard the isentropic flow of the fluid is viscosity, and this occurs near the nozzle walls only. Therefore, evaluation of the boundary layer in the nozzle provides all that is required for calculation of the velocity coefficient. The parameter of interest is the boundary-layer momentum thickness, which is a measure of the freestream momentum lost due to viscosity. The momentum thickness is defined as

$$\theta = \int_{y=0}^{\delta} \frac{\rho V}{\rho_\infty V_\infty}\left(1 - \frac{V}{V_\infty}\right)\mathrm{d}y \tag{35}$$

The relationship between the momentum thickness and the velocity coefficient is

$$C_V = 1 - (4\theta/D) \text{ or } C_{V9} = 1 - (4\theta_9/D_9) \tag{36}$$

This can be derived from the continuity equation, the definition of velocity coefficient, and the assumption that the boundary-layer displacement thickness δ^* is much smaller than the nozzle diameter.

A computer program was used to calculate the boundary-layer momentum thickness based on the method of Ref. 3. This reference examines several well-known procedures for calculating the boundary-layer characteristics and correlates the results as functions of freestream Mach number and Reynolds number based upon an equivalent flat-plate length. Using this boundary-layer computer program, the velocity coefficient was calculated from Eq. (36) by using the calculated momentum thickness at the nozzle exit. The C_{V9} thus represents the cumulative effect of the boundary layer/viscosity for the entire nozzle. The loss due to the boundary layer is therefore

$$\Delta C_{fg} = 1 - C_{V9} \tag{37}$$

and this is subtracted from the $C_{fg\text{inviscid}}$ calculated in Eq. (32). Since $C_{fg\text{theory}}$ and C_{v9} are very near 1.0

$$C_{fg\text{theory}} = C_{fg\text{inviscid}} - (1 - C_{V9}) \cong C_{V9} C_{fg\text{inviscid}} \tag{38}$$

Comparison of Theory with Experiment

The theoretical approach was tested by comparing calculated thrust coefficient results with two conical CD nozzle scale-model tests. The exact model test geometries and flow conditions were programmed for the STC analysis, and thrust coefficients were calculated as previously discussed.

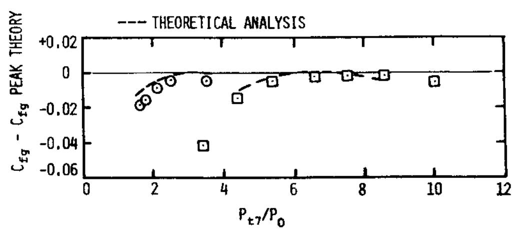

Fig. 5 Thrust coefficient theory vs model test. Model test, $A_9/A_8 = 1.027$; $A_7/A_8 = 2.52$, $L/D_8 = 1.64$, $\alpha = 0.5°$. Model test, $A_9/A_8 = 1.559$, $A_7A_8 = 1.39$, $L/D_8 = 1.97$, $\alpha = 7.5°$.

The model test thrust coefficients are shown in Fig. 5 as a difference from the peak thrust coefficient calculated from the theoretical analysis. These results indicate that the thrust coefficient of the theoretical analysis is within 0.3% of the scale-model test results. Based on these results, it was concluded that the theoretical analysis will provide reasonably accurate estimates of nozzle thrust coefficient (within $\sim \pm 1/4\%$).

Theoretical Analysis of Nozzle Entrance Temperature Profiles

Having established the accuracy of the theoretical approach, the technique was used to analyze conical CD nozzle performance with nozzle entrance total temperature distortion. Two temperature profiles were analyzed; one was typical of a nonafterburning subsonic cruise condition, and the other represented an extreme temperature profile typical of a part-afterburning condition, as shown in Fig. 6. The nozzle geometries typically were consistent with the operating condition. Each geometry was evaluated first with the distorted nozzle entrance conditions and then with uniform profiles and temperature levels that were approximately the average of the distorted cases. (Entrance pressure was unchanged.)

Results of this analysis are shown in Fig. 7. For both the subsonic cruise and the part-reheat comparisons, the calculated thrust coefficients for the uniform and distorted profiles are within $\sim 0.1\%$ of each other. Note that, when the temperature profile is mass weighted, the thrust coefficients appear to be low, as expected from the analysis of the averaging procedures. These results indicate that CD nozzle

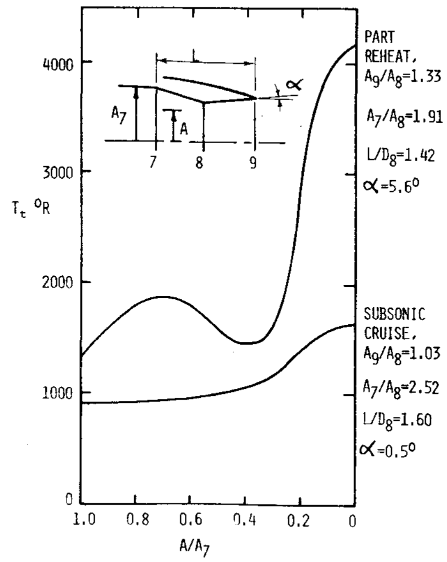

Fig. 6 Temperature profiles for theoretical analysis.

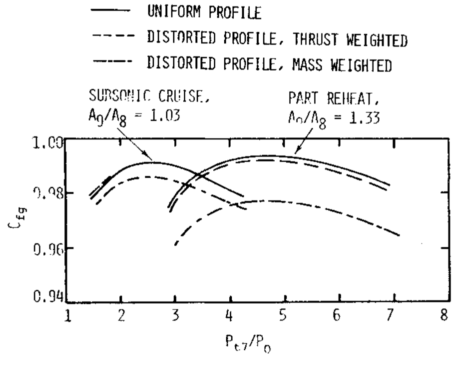

Fig. 7 Comparison of theoretical analysis thrust coefficients, uniform vs distorted entrance temperature profiles.

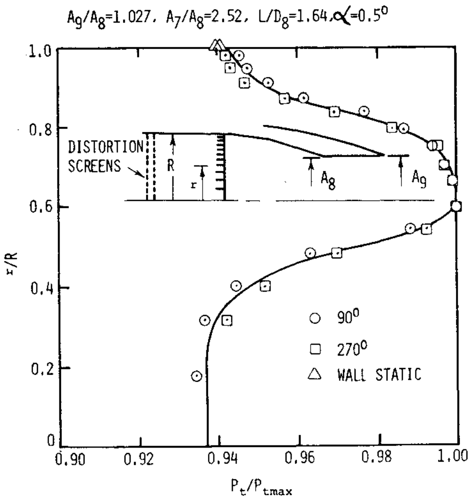

Fig. 8 Radius ratio r/R vs pressure distortion $P_t/P_{t\max}$; nozzle entrance pressure profile for model distortion test.

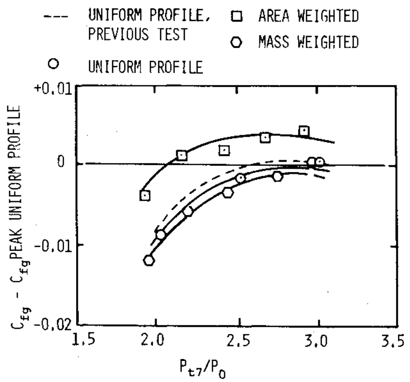

Fig. 9 Model test thrust coefficient, uniform vs distorted pressure profile.

thrust coefficients will be unaffected by the presence of nozzle entrance total temperature distortion, within the limits investigated, provided that the profiles are averaged properly. Additionally, comparison of the nozzle wall pressure distributions and nozzle exit plane flow angle and Mach number indicated that the nozzle flowfield aerodynamics are very similar between uniform and distorted temperature profiles.

Experimental Analysis of Nozzle Entrance Pressure Profiles

A scale-model test of a low-area-ratio conical CD nozzle was conducted in an attempt to determine the effects of a total pressure profile on nozzle performance. Both the distorted profile, shown in Fig. 8, and a uniform profile were tested in order to obtain a direct comparison of performance.

Measured thrust coefficients are presented in Fig. 9 for both the flat-profile and distorted-profile cases. This same model had been tested previously with no profiles in a different test program, and the results of that test also are shown in Fig. 9. Comparison with the flat-profile results indicated excellent repeatability, within 0.1%.

The nozzle performance results were calculated by both mass weighting and area weighting to determine the ideal thrust. As expected, the area-weighted approach gave a lower entrance pressure than the mass-weighted pressures, thereby making the thrust coefficients appear higher.

Comparison of the mass-weighted profile with the flat profile results shows agreement within demonstrated repeatability of the test facility. It was concluded that total pressure profiles of this magnitude, when properly averaged, do not adversely effect CD nozzle performance.

Summary and Conclusions

Based on the results of this study, these conclusions were reached:

1) Mass weighting of nozzle entrance total pressure profiles and "thrust weighting" of entrance total temperature profiles provide the most practical and correct averaging method for analyzing full-scale engine nozzle performance.

2) Total temperature profiles (as high as 65%) have a negligible effect (less than 0.2%) on CD nozzle performance.

3) Total pressure profiles less than 8% (max-min) have a negligible effect (less than ~0.1%) on CD nozzle performance.

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